Chapter 6

Voyage to the Future

Before turning to the subject of gravity and the curvature of spacetime, let us describe *your* recent trip to the center of our galaxy, which is some 30,000 light-years from the Sun. For this purpose we shall first study what might be called *uniform accelerated motion.*

Almost everyone knows that the acceleration of falling objects near the earth's surface is 32 ft/sec^2 or 9.8 m/sec^2, but few have noticed that this is just a trifle less than 1 light-year/year^2. Thus, if one accelerates from rest at 1g, one will, according to seventeenth century physics, reach the speed of light at the end of one year, while at the end of ten years one will be traveling at about ten times the speed of light. The familiar formula $s = \frac{1}{2}at^2$ tells us that one will cover a distance of approximately 50 light-years during this ten-year period. If one spends a similar period of time decelerating at 1g, then one will reach a point about 100 light-years from earth before one comes to rest. Although this is a long distance, it is still only 1/300th the distance to the galactic center. Since Einstein's theory precludes accelerating up to or beyond the speed of light, it would seem that galactic scale space travel is out of the question; but you actually went there, so how did you do it?

When we refer to uniform acceleration, we shall be referring to the acceleration relative to a fixed inertial reference frame, e.g., the frame with respect to which the galactic center is essentially at rest. If, in the absence of other forces, the floor pushes upward upon you with a force F, and your rest mass is m, you experience an upward acceleration $a = F/m$. Of course, $F = dP/dt$, where t is the time measured in the chosen inertial frame, and P is the momentum, which we have seen is related to the velocity $v = dx/dt$ by

$$P = m\frac{dx}{d\tau} = \frac{mv}{\sqrt{1-(v/c)^2}}.$$

Here τ is the proper time, as measured by a clock attached[1] to the object that is undergoing the acceleration. Thus,

$$\frac{d}{dt}\left[\frac{v}{\sqrt{1-(v/c)^2}}\right] = a.$$

If the acceleration a is independent of t, we can integrate this equation to obtain

$$\frac{v}{\sqrt{1-(v/c)^2}} = at + k,$$

where k is a constant of integration that can be eliminated by setting the clock appropriately. The solution of the equation is then

$$v = \frac{at}{\sqrt{1+(at/c)^2}}.$$

Substituting $v = dx/dt$ and integrating again, we obtain

$$ax/c^2 = \sqrt{1+(at/c)^2} - 1, \tag{6.1}$$

where the constant of integration has again been eliminated by selecting the origin appropriately. When $at/c \ll 1$, these equations reduce to their Newtonian counterparts $v \approx at$ and $x \approx \frac{1}{2}at^2$. On the other hand, when $at/c \gg 1$, we get $v \approx c$ and $x \approx ct$. When $at/c \approx 1$, $v \approx 0.7c$ and $ax/c^2 \approx 0.4$. For example, if the acceleration $a = 32$ ft/sec^2 and $c = 186,000$ mi/sec, one finds $c/a \approx 1$ year. Therefore, when you accelerated at 1g for approximately 1 year, you actually ended up traveling at 70 percent of the speed of light and had traveled a total distance of 0.4 light-years. That is about one-tenth the distance from the Sun to the nearest other star.

Once you began traveling at relativistic speeds, you had to distinguish between the time t measured in a fixed inertial frame, and the proper time τ measured on your spaceship. These are related by

$$\frac{d\tau}{dt} = \sqrt{1-(v/c)^2} = \frac{1}{\sqrt{1+(at/c)^2}}. \tag{6.2}$$

[1] One can avoid having to tackle any question concerning whether or not the behavior of the clock is affected by acceleration, the answer to which would depend upon the construction of the clock, by employing for a short interval a clock that is moving at uniform speed and happens to be moving (for the time being) at approximately the same speed as the body undergoing acceleration.

$a\tau/c$	at/c	ax/c^2	v/c
0.0	0.00	0.00	0.000
1.0	1.18	0.54	0.762
2.0	3.63	2.76	0.964
3.0	10.0	9.1	0.995
4.0	27.3	26.3	0.999
5.0	74	73	1.000
6.0	202	201	1.000
7.0	548	547	1.000
8.0	1490	1489	1.000
9.0	4052	4051	1.000
10.0	11,013	11,012	1.000
10.3	14,866	14,865	1.000

Table 6.1: Uniform Acceleration

Integrating this equation, we obtain

$$a\tau/c = \ln\left[(at/c) + \sqrt{1+(at/c)^2}\right],$$

where the constant of integration has been eliminated by setting the shipboard clock appropriately. We note, in particular, that when $at/c \ll 1$, one has $\tau \approx t$, while when $at/c \gg 1$, one has

$$a\tau/c \approx \ln(2at/c).$$

This logarithmic dependence of the proper time upon the inertial time is highly significant for the feasibility of long distance space travel and of time travel into the future.

All the equations we have been discussing can be reexpressed in terms of the proper time τ. One finds that

$$\begin{aligned} at/c &= \sinh(a\tau/c), \\ ax/c^2 &= \cosh(a\tau/c) - 1, \\ v/c &= \tanh(a\tau/c). \end{aligned}$$

In the accompanying table are shown these three quantities for various values of $a\tau/c$.

Bearing in mind that for an acceleration of 1g, one has $c/a \approx 1$ year, we see that after 3 years acceleration at 1g you traveled about 10 light-years, and were traveling at 99.5 percent of the speed of light. We also see from the table that after approximately 10.3 years, the spaceship had traveled about 15,000 light-years, which is half the way from the Sun to the galactic center. At this point you were traveling at a speed that is just 2 ft/sec shy of the speed of light, and you began slowing down. After another 10.3 years of deceleration at 1g, you reached the vicinity of the galactic center. After spending some time studying the region, you began your 20.6 year return journey. When you arrived back at earth, earth history had evolved over 60,000 years while you had aged a little more than 40 years.[2]

Ex. 10 *We have made believe that the mass of your spaceship would stay the same throughout the voyage, but this is far from true, even if you were to use photon propulsion. After looking up "the rocket equation" in an elementary mechanics textbook, devise a relativistic analog of that equation, and use it to analyze anew your trip to the galactic center.*

Ex. 11 *Estimate the fuel requirements for your trip.*

A Curious Alternative Interpretation

There is an interesting alternative way to interpret Eq. (6.1). After using Eq. (6.2) to reexpress the right side of Eq. (6.1) in terms of v, multiply both sides of the resulting equation by Mc^2, thereby obtaining

$$\mathrm{KE} + \mathrm{PE} = 0,$$

where

$$\mathrm{KE} = \frac{Mc^2}{\sqrt{1-(v/c)^2}} - Mc^2$$

is the relativistic kinetic energy of an object of rest mass M and speed v, while

$$\mathrm{PE} = -Max$$

is the potential energy of an object of rest mass M at $-x$ in a gravitational field of strength Ma. This suggests that you might adopt the egocentric

[2]After working out the details of this voyage, I learned that Kip Thorne had contemplated the same voyage in his book *Black Holes & Time Warps* (Norton, New York, 1994).

viewpoint that your spaceship, supported by its engines, has been at rest all the while, and that the rest of the galaxy has been in free fall in a uniform gravitational field.

If you were to adopt this viewpoint, how would you explain the fact that when your journey was completed the earth had aged 60,000 years while you had aged only 40 years? The usual time dilation formula would imply that the earth, moving at relativistic speeds, aged *less* than you did.

Later we shall see that an explanation of this paradox is to be found in the fact that in the presence of a gravitational field, the rate of a clock depends not only upon its motion but also upon the gravitational potential where it is located. A clock at a lower gravitational potential ticks more slowly than does a clock at a higher gravitational potential.

Epilogue

You left earth toward the end of that great era of materialism that was born of the industrial revolution in the eighteenth century and which matured as "consumerism" following the technological revolution of the twentieth century, when the only measure of goodness, truth and beauty was a dollar value in the market place. To believe in any more fundamental measure of goodness, truth or beauty was to engage in an anti-popular "elitism." In this, the heyday of marketing, that was good, true or beautiful that was deemed to be such by the greatest number of consumers, reacting, as earlier did Pavlov's dogs to a dinner bell, to a constant barrage of advertising that was designed to create insatiable desires.

Where once priests, scientists and artists labored because they loved what they were doing, a tawdry commercialism now pervaded all aspects of life, while Ethics, the Sciences and the Arts languished.[3]

The world was preoccupied with having fun, and almost all workers were employed in some aspect of the entertainment business. Bereft of real education, these workers rarely experienced the thrill of having a new idea. Instead they relived the past, remaking old movies and games using the lat-

[3]These were replaced by the ethics, science and art businesses, which assessed goodness, truth and beauty by the sole criterion of marketability, and which were as remote from their respective namesakes as the education business of the late twentieth century had been from Education. During the misologistic nineties logical proof and mathematical analysis became anachronisms, replaced by the notion of popular concensus. The last university closed its doors in the twenty-first century, after most students realized they could get an equivalent education directly off the Web.

est technological gimicry.[4] A world weaned on a cascade of sight and sound bites had little patience with actual reading, or with the thoughtful reflection that such reading might inspire. Physical libraries decayed, and many of the books they once contained were lost forever.

By the end of the second millennium the national governments that had become so strong following the industrial revolution were already in decline. The center of commerce and industry continued to move inexorably westward, beyond America, as those who put no national interest above their own personal self-interest left the inhabitants of the "western democracies" to pay off debts that had been incurred over many years of deficit financing, while corporate and political "leaders" moved on to bring the democracy business to new demesnes that were ripe for exploitation.

Since rich corporations could better afford to maintain security forces than the beleaguered national governments, each of the most successful businesses developed its own militia, which, in the absence of governmental oversight, was employed routinely to enhance its competitive advantage. Feeling that their wealth and their privileges were threatened by the corporate anarchy that swept the world in the wake of the great financial collapse that occurred early in the new millennium, the most powerful individuals finally united to establish the first effective world government, Earth, Inc.[5]

Fundamentalists of all persuasions, agreeing only that there should be more to life than those bread and circuses that were hawked by Earth, Inc., bristled under hyperconsumerism, but they were unable to keep from one another's throat long enough to put up an effective resistance to the systematic corruption of the human spirit and the overconsumption of Nature's bounty.

In spite of the leaders' best efforts to take care of excess population[6] by the traditional method of inciting into an irrational frenzy the passions of one such group of expendables against another, world population continued to grow exponentially and, in the interest of human "needs," many forms of nonhuman plant and animal life were driven to extinction, either by direct human predation or by the destruction of habitat. Eventually the ever-growing number of consumers discovered that Nature had some rather brutal tricks up her sleeves. By delaying the inevitable "correction," humans only transformed what might have been a relatively gentle rebuke into a massive

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[4]The hundredth remake of *Gone with the Wind* was no more memorable than the ninety-nine that preceeded it, every vestige of significance having been long ago replaced by form.

[5]Of course, voting rights were determined by how many shares of stock one owned.

[6]We refer here to those who were, for whatever reason, unable or unwilling to consume that which was peddled by Earth, Inc.

liquidation.

Ex. 12 *Show that if Nature were to permit world population to continue growing at the same average per capita rate as it has since the industrial revolution, doubling every fifty years or so, by the year 3000 people would be standing shoulder to shoulder, covering the entire land mass of the earth.*

While the earth can no longer support large animals, life is far from absent almost 60,000 years and several ice ages after the cataclysm. One key to the success of the inheritors of the earth was their use of the fruits of science, guided by a profound knowledge of history, to tame those cycles of despotism and anarchy that characterized human history and to maintain a balance between the demands of head and heart, of reason and emotion. Another was the development of economic and educational support systems that fostered not greed, self-aggrandizement and violence, but rather a passion for that spiritual, artistic and mental growth that constitutes truly enlightened self-interest.