Chapter 1

Inertial Frames and the Principle of Relativity

There is little in everyday experience that prepares one for the concept of an *inertial frame of reference*. One grows accustomed to things slowing down and stopping when that which powers them, be it human or animal muscle, steam, gasoline, diesel fuel or electricity, ceases to do so. Indeed, even in a popular television show it is the speed of the starship and not its acceleration that is determined by how hard the engines are working. The starship invariably slows down and “stops” when its engines are shut off. While such television shows have adopted a great deal of the jargon of modern science, they have not begun to incorporate the content of seventeenth century physics, let alone that of the twentieth century. One can only imagine how much more educational and even how much more exciting such shows might be if they successfully depicted those elusive concepts upon which the physics of the seventeenth through twentieth centuries is based.

Even more imagination was required in the seventeenth century than is required today to conceive of the complete elimination of friction, and to infer that **in the absence of all forces, including friction and gravity, objects move at constant speed in straight lines.** Even if a seventeenth century physicist had been able effectively to eliminate friction, he would not have been able to turn off the ever-present gravitational field that bound him pretty securely to the earth’s surface. Isn’t it fortunate that nature allows one to infer something about motion in general from a study of motion confined to a horizontal plane?

Moreover, someone eventually had to relinquish an egocentric perspective long enough to consider the questions “what if one of the moving objects were itself a sentient being” and “what if that observer were to ask the same question concerning the motion of objects that are not subject to any forces?" Would that observer be able to confirm the same result that, relative to him, objects that are subject to no net force move at constant speed in straight lines? The truth of the guess that he would, in fact, do so was accepted more on the basis of its reasonableness, within the context of prevailing philosophical notions, than upon any reproducible experimental test.

The experimental confirmation of the existence of inertial frames of reference, with respect to which objects subjected to no net force move at constant speed in straight lines, is possible only to the degree to which forces can be turned off. Today we can test the hypothesis more completely than could seventeenth century physicists, because we have acquired the ability to reduce greatly the effects of friction. We are even able to reduce the effect of gravitational forces by performing experiments in space. Nevertheless, there will always remain small deviations from straight line motion at constant speed. What is important is being able reasonably to attribute the observed deviations to identifiable forces that the experimenter has, for technical reasons, not yet been able to eliminate.

In this way one can formulate provisional truths concerning the way things are, upon which provisional truths additional hypotheses can be formulated and subjected to testing, until eventually one has assembled an impressive body of provisional knowledge, all linked together by the requirement of logical consistency.

Yet, with the development of more accurate experiments, or the performance of a new experimental test, there always remains the possibility that the grand edifice will be found to be flawed. **Scientific truth is always provisional; only error can be verified absolutely**.

The *principle of relativity* was already quite an old concept when in 1905 Albert Einstein made it the cornerstone of his bold new theory. As he himself pointed out in his book, *The Meaning of Relativity*, the concept of relativity was an important element of seventeenth century Newtonian mechanics.[1] Stated quite simply, the idea was that **the basic laws of physics[2] assume the same form in all inertial frames of reference**. According to this doctrine, observers in two different inertial frames can, with equal right and equal success, employ the same basic laws of physics relative to their own frame of reference.

In the nineteenth century the laws governing electromagnetic phenomena

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[1] A. Einstein, *The Meaning of Relativity*, fifth edition, (Princeton University Press) 1956.

[2] e.g., Newton's three laws of motion and his law of gravitation.

were formulated, at first in a piecemeal fashion by such people as Faraday and Ampère, and later in a consistent integrated manner by Maxwell. The Maxwell theory, with its novel idea of the displacement current, implied the possibility of electromagnetic waves, the actual existence of which was established by Hertz several decades later. Of course, light itself was an example of such waves, albeit of exceedingly high frequency.

It is important to appreciate that the Maxwell theory was not just a qualitative theory. Instead, it consisted of a set of partial differential equations governing electric and magnetic fields, and the predictions were quite specific. Thus, for example, the speed of propagation of electromagnetic waves was predicted by the theory in terms of parameters determined by electrostatic and magnetostatic measurements. The predicted speed agreed well with the known speed of light. Furthermore, the existence of two states of polarization of the waves was predicted by the theory, and light was already known to have that property. In this and in many other respects the Maxwell theory of electromagnetism was, like Newton's theory of gravitation, an extremely successful theory.

However, the Maxwell theory appears not to respect the principle of relativity. If one writes out the Maxwell field equations in one frame of reference, and then transforms the equations to a second frame of reference using the Galilean transformation formula

$$\vec{x} = \vec{x}' + \vec{v}t,$$

under which the Newtonian theory is invariant, the Maxwell equations take on a different form in the new primed reference system. Furthermore, there is no way to transform the electric and magnetic fields that will result in equations identical in form to the original ones. The theory is simply not invariant under Galilean transformations!

In more graphic terms the situation is this. The Maxwell theory predicts that light waves travel at a certain speed c in all directions. If one transforms to another frame of reference moving at speed v relative to the first, then the speed of light, it would seem, should lie between $c - v$ and $c + v$, depending upon the direction the light travels. Thus, it would appear that electromagnetic phenomena provide a means for selecting from among all inertial frames one special one, with respect to which the speed of light is c in every direction. Only in this special frame would the Maxwell equations be valid.

Toward the end of the nineteenth century numerous attempts were made to demonstrate the "aether drift." The most famous were the experiments of Michelson and Morley. These were experiments designed to measure the speed of the earth relative to the frame of reference in which the speed of light is c in every direction. Borrowing a phrase from Newton, let us refer to this special frame of reference as the "primary inertial frame."

If, momentarily, the earth happened to be at rest with respect to the primary inertial frame, it would certainly not remain at rest, for six months later, having moved half way around its orbit about the sun, it would be moving at twice the earth's orbital speed of 30 km/sec with respect to the primary inertial frame. For this reason it is important to repeat the experiment several times during the year.

Incredibly, Michelson and Morley found no evidence for any aether drift at *any* time of year. The speed of light seemed to be c in every direction, no matter when one measured that speed.[3]

This state of affairs gave rise to many bizarre attempts to comprehend what had gone wrong with classical physics. It was Einstein, however, who cut away all the irrelevant considerations and put his finger on what was really wrong.

People had been too willing to admit that the Maxwell theory of electromagnetism does not agree with the principle of relativity. The Maxwell theory *is* invariant under a group of transformations that we call the Lorentz group, but this is not the same as the Galilean group of Newton's theory!

If one wishes to have the Maxwell equations valid in all inertial frames, all one need do is admit that the Galilean transformations are not the correct transformations for going from one inertial frame to another. Instead, the correct transformations are those which leave the Maxwell equations unchanged.[4]

The admission that the various inertial frames are connected by the Lorentz transformations rather than the Galilean transformations has profound implications. First of all, it means that Newtonian mechanics is not quite correct, insofar as it is a theory based upon Galilean invariance, not Lorentz invariance. The only reason Newtonian mechanics worked so well for so long is that for most situations considered in practice, the speeds of bodies are much less than the speed of light. Of course, twentieth century particle experiments have shown that when speeds approach c, Newtonian physics does break down in just the way that was predicted by Einstein's theory.

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[3] A nice description of the Michelson-Morley experiments is given by P. G. Bergmann, *Introduction to the Theory of Relativity* (Prentice-Hall) 1942.

[4] A. Einstein, *Zur Elektrodynamik bewegter Körper*, Annalen der Physik **17**, 1905. A translation can be found in *The Principle of Relativity* by A. Einstein et al. (Dover) 1923.

What was very hard for people to swallow back in 1905 was the idea that space and time are not *absolute*. According to the Einstein theory an object which is moving relative to an observer with speed v suffers a *Fitzgerald contraction* $\sqrt{1 - v^2/c^2}$ along the direction of relative motion. Two identical twins who pass each other would claim that the other had shrunk somewhat, although neither would admit having changed at all.[5] Similarly, if the twins wore identical watches, each would claim the other twin's watch ticks more slowly by a factor $\sqrt{1 - v^2/c^2}$ than his own, the phenomenon of *time dilation*. Finally, if object B moves with speed v_{AB} relative to object A, while object C moves with speed v_{BC} relative to object B (directly away from object A), the speed of C relative to A is not simply $v_{AB} + v_{BC}$ but rather is given by the *velocity addition formula*

$$v_{AC} = \frac{v_{AB} + v_{BC}}{1 + v_{AB}v_{BC}/c^2}.$$

Incidentally, note that if body C is replaced by a light ray, then the speed of this light ray is c relative to body A and c relative to body B as well. Just plug in $v_{BC} = c$ and see what happens!

Our immediate objective is to acquire some familiarity with the Lorentz transformations, and to see what they imply about space and time and the laws of nature. Before we leap into the complexities of Lorentz transformations, we shall consider certain simpler groups of transformations just for practice. It will also be necessary to review some of the basic notions of the Maxwell theory of electromagnetism, so that we can fully appreciate how the Lorentz transformations emerge naturally from that theory.

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[5] We are not talking here about visual appearance, which would require a deeper analysis involving a study of the propagation of light rays from object to observer.