The Axis Data of Known Stationary Axisymmetric Solutions

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Note that the domain of the axis data is a proper subset of the whole axis if $f(z,0) := \mathrm{Re}\mathcal{E}(z,0) + |\Phi(z,0)|^2$ vanishes for any real value of z. I shall assume that $z = +\infty$ is in the domain.

In the case when the axis data $\mathcal{E}(z,0)$ and $\Phi(z,0)$ are rational functions of z, I shall classify the data using two integers, the first of which is the highest power of z in the denominator of $\mathcal{E}(z,0)$ and the second of which is the highest power of z in the denominator of Φ. In the case of papers that are listed in Vladimir Manko's *List of Publications*, I shall provide that number as well.

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1. Herrera & Manko, Phys. Lett. A **167**, 238 (1992): [3,0] M31

$$\mathcal{E}(z,0) = 1 - \frac{2m}{z} + \frac{8im^3\alpha}{z^3}, \quad \Phi(z,0) = 0.$$

2. Manko & Sibgatullin, Class. Quant. Grav. **9**, L87 (1992): [1,2] M32

$$\mathcal{E}(z,0) = \frac{z-m-ia}{z+m-ia}, \quad \Phi(z,0) = \frac{ib}{(z+m-ia)^2}.$$

3. Manko & Sibgatullin, Phys. Lett. A **168**, 343 (1992): [2,2] M33

$$\mathcal{E}(z,0) = 1 - \frac{2m}{z+m} + \frac{2ia}{(z+m)^2}, \quad \Phi(z,0) = \frac{q}{z+m} + \frac{ib}{(z+m)^2}.$$

Note the possible substitution $z + m \to z$.

4. Manko & Sibgatullin, PRD **46**, 4122 (1992): [4,2] M35

$$\mathcal{E}(z,0) = 1 - \frac{2m}{z+m} + \frac{4i\alpha m^2}{(z+m)^2} - \frac{16m^4\beta^2}{(z+m)^4}, \quad \Phi(z,0) = \frac{4im^2\beta}{(z+m)^2}.$$

   Note the possible substitution $z+m \to z$.

5. Manko & Sibgatullin, JMP **34**, 170 (1993): [1,2] M36

$$\mathcal{E}(z,0) = \frac{z-m-ia}{z+m-ia}, \quad \Phi(z,0) = \frac{q}{z+m-ia} + \frac{ib}{(z+m-ia)^2}.$$

6. Aguirregabiria, Chamorro, Manko and Sibgatullin, PRD **48**, 622 (1993): [3,2] M40

$$\mathcal{E}(z,0) = 1 - \frac{2m}{z+m} - \frac{2ia}{(z+m)^2} - \frac{2c}{(z+m)^3}, \quad \Phi(z,0) = \frac{ib}{(z+m)^2}.$$

   Note the possible substitution $z+m \to z$.

7. Manko, Phys. Lett. A **181**, 349 (1993): [1,2] M44

$$\mathcal{E}(z,0) = \frac{z-m-ia}{z+m-ia}, \quad \Phi(z,0) = \frac{q+ib}{z(z+m-ia)}.$$

8. Manko, Class. Quant. Grav. **10**, L239 (1993): [1,2] M45

$$\mathcal{E}(z,0) = \frac{z-m-ia}{z+m-ia}, \quad \Phi(z,0) = \frac{ib}{z(z+m-ia)}.$$

9. Manko, Martín, Ruiz, Sibgatullin and Zaripov, PRD **49**, 5144 (1994); Manko, Martín and Ruiz, PRD **49**, 5150 (1994): [2,2] M47

$$\mathcal{E}(z,0) = \frac{z(z-m-ia)+b}{z(z+m-ia)+b}, \quad \Phi(z,0) = \frac{qz+ic}{z(z+m-ia)+b}.$$

10. Manko, Martín and Ruiz, Phys. Lett. A **196**, 23 (1994): [2,0] M49

$$\mathcal{E}(z,0) = \frac{z-k-m-ia}{z-k+m-ia} \cdot \frac{z+k-m-ia}{z+k+m-ia}, \quad \Phi(z,0) = 0.$$

11. Manko, Martín and Ruiz, JMP **36**, 3063 (1995): [2,2] M54

$$\begin{aligned}
\mathcal{E}(z,0) &= \frac{(z-m-ia)(z+ib)-k}{(z+m-ia)(z+ib)-k}, \\
\Phi(z,0) &= \frac{qz+ic}{(z+m-ia)(z+ib)-k}.
\end{aligned}$$

An alternative parametrization is given by the following:

$$\begin{aligned}
\mathcal{E}(z,0) &= \frac{z-k'-m'-i(a'+\nu')}{z-k'+m'-i(a'-\nu')}\cdot\frac{z+k'-m'-i(a'-\nu')}{z+k'+m'-i(a'+\nu')}, \\
\Phi(z,0) &= \frac{2(q'z+ic')}{[z-k'+m'-i(a'-\nu')][z+k'+m'-i(a'+\nu')]},
\end{aligned}$$

where

$$\begin{aligned}
&m = 2m', \quad a = a' + k'\nu'/m', \quad -a' + k'\nu'/m', \\
&k = k'^2 - m'^2 - \nu'^2 + k'^2\nu'^2/m'^2, \quad q = 2q', \quad c = 2c'.
\end{aligned}$$

12. Manko, Martín and Ruiz, Phys. Rev. D **51**, 4187 (1995): [2,2] M52

$$\mathcal{E}(z,0) = \frac{z^2+a_1z+a_2}{z^2+b_1z+b_2}, \quad \Phi(z,0) = \frac{c_1z+c_2}{z^2+b_1z+b_2}.$$

Here the six constants are complex.

13. Ruiz, Manko and Martín, Phys. Rev. D **51**, 4192 (1995): [N,N] M53

$$\mathcal{E}(z,0) = \frac{z^N+\sum_{l=1}^N a_l z^{N-l}}{z^N+\sum_{l=1}^N b_l z^{N-l}}, \quad \Phi(z,0) = \frac{\sum_{l=1}^N c_l z^{N-l}}{z^N+\sum_{l=1}^N b_l z^{N-l}}.$$

Here the $3N$ constants are complex. The solution is given in terms of determinants.

14. Manko and Ruiz, GRG **29**, 991 (1997): [2,2] M61

$$\begin{aligned}
\mathcal{E}(z,0) &= \frac{z^2-2(m+ia)z+m^2-a^2-c^2}{z^2+2(m-ia)+m^2-a^2-c^2}, \\
\Phi(z,0) &= \frac{2ic'}{z^2+2(m-ia)+m^2-a^2-c^2},
\end{aligned}$$

where

$$c^2 := \frac{c'^2}{m^2-a^2}.$$

15. Manko and Ruiz, Class. Quant. Grav. **15**, 2007 (1998): [N,0] M64

$$\mathcal{E}(z,0) = 1 + \sum_{l=1}^{N} \frac{e_l}{z - \beta_l}, \quad \Phi(z,0) = 0,$$

or, alternatively,

$$\mathcal{E}(z,0) = \frac{z^N + \sum_{l=1}^{N} a_l z^{N-l}}{z^N + \sum_{l=1}^{N} b_l z^{N-l}}, \quad \Phi(z,0) = 0.$$

The relation between the e_l and the α_n and β_l is

$$e_l = \frac{2\Pi_{n=1}^{2N}(\beta_l - \alpha_n)}{\Pi_{k \neq l}(\beta_l - \beta_k)\Pi_{k=1}^{N}(\beta_l - \bar{\beta}_k)}.$$

16. Manko, Mielke and Sanabria-Gómez, PRD **61**, 08150 (2000): [2,2] M72

$$\begin{aligned}\mathcal{E}(z,0) &= \frac{(z-m-ia)(z+ib)+d-\delta-ab}{(z+m-ia)(z+ib)+d-\delta-ab}, \\ \Phi(z,0) &= \frac{i\mu}{(z+m-ia)(z+ib)+d-\delta-ab},\end{aligned}$$

where

$$\delta := \frac{\mu^2 - m^2 b^2}{m^2 - (a-b)^2}, \quad d := \frac{1}{4}\left[m^2 - (a-b)^2\right].$$

17. Manko and Sanabria-Gómez, PRD **62**, 044048 (2000): [2,2] M74

$$\begin{aligned}\mathcal{E}(z,0) &= \frac{(z-m-ia)(z+ib)+d-\delta-ab}{(z+m-ia)(z+ib)+d-\delta-ab}, \\ \Phi(z,0) &= \frac{qz+i\mu}{(z+m-ia)(z+ib)+d-\delta-ab},\end{aligned}$$

where

$$\delta := \frac{\mu^2 - m^2 b^2}{m^2 - (a-b)^2 - q^2}, \quad d := \frac{1}{4}\left[m^2 - (a-b)^2 - q^2\right].$$

18. Manko and Sanabria-Gómez, Phys. Lett. A ???: [2,2]

$$\begin{aligned}\mathcal{E}(z,0) &= \frac{(z-2m-2ia)(z+2ib)+m^2-(a+b)^2-\sigma^2}{(z+2m-2ia)(z+2ib)+m^2-(a+b)^2-\sigma^2}, \\ \Phi(z,0) &= \frac{c^2-4m^2b^2}{(z+2m-2ia)(z+2ib)+m^2-(a+b)^2-\sigma^2},\end{aligned}$$

where

$$\sigma^2 := \frac{c^2 - 4m^2b^2}{m^2 - (a-b)^2}.$$